Or, completing the multiplications,

$$(y^3+z^3-b^3)^3+27b^3y^3z^3=0.$$

Restoring the original values of y, z, and b, we get,

$$27c(x^2-a^2)=(2a-c)^3.$$

Hence $x=\sqrt{\left(\dfrac{(2a-c)^3+27a^2c}{27c}\right)}$.

This may be the root of the given equation or the root of any of the assumed equations, depending on the various values of a and c.

[This example is found in Bonnycastle's Algebra (1845), page 97.]
Also solved by *P. S. BERG*, *H. C. WILKES*, and *G. B. M. ZERR*.

71. Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

When $x=0$, find the limit of the expression

$$u=\left(\frac{m+x}{m-x}\right)^{\frac{1}{x}}+\left(\frac{m-x}{m+x}\right)^{\frac{1}{x}}.$$

I. Solution by O. W. ANTHONY, M. Sc., Columbian University, Washington, D. C., and G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

Let $u=u_1+u_2$. $\therefore$ $u_1=\left(\dfrac{m+x}{m-x}\right)^{\frac{1}{x}}$,

$$\log u_1=(1/x)\{\log(m+x)-\log(m-x)\}$$

$$=(1/x)\{[\log m+(x/m)-(x^2/2m^2)+(x^3/3m^3)-\ldots\ldots]-$$

$$[\log m-(x/m)-(x^2/2m^2)-(x^3/3m^3)-\ldots\ldots]\}$$

$$=\frac{2}{x}\left(\frac{x}{m}+\frac{x^3}{3m^3}+\frac{x^5}{5m^5}+\ldots\ldots\right)=2\left(\frac{1}{m}+\frac{x^2}{3m^2}+\frac{x^4}{5m^5}+\ldots\ldots\right)$$

$=2/m$, when $x=0$. $\log u_2=-\log u_1=-2/m$, when $x=0$.

$\therefore$ $u_1=e^{2/m}$, $u_2=e^{-2/m}$. $\therefore$ $u=e^{2/m}+e^{-2/m}$ when $x=0$.

II. Solution by H. C. WHITAKER, M. Sc., Ph. D., Professor of Mathematics in Philadelphia Manual Training School, Philadelphia, Pennsylvania.

Since $(1+x)^{1/x}=e$ when $x=0$, we have

$$\left(\frac{m+x}{m-x}\right)^{\frac{1}{x}}=\left[\left(1+\frac{2x}{m-x}\right)^{\frac{m-x}{2x}}\right]^{\frac{2}{m-x}}=e^{2/m} \text{ when } x=0.$$

In the same way $\left(\frac{m-x}{m+x}\right)^{\frac{1}{x}}=e^{-2/m}$ when $x=0$. Hence $u=e^{2/m}+e^{-2/m}$.

Also solved by *J. SCHEFFER.*

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

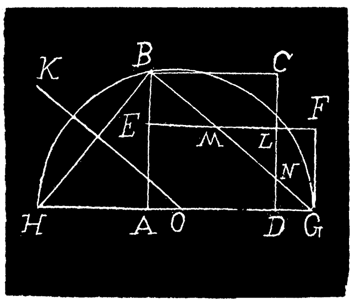
SOLUTIONS OF PROBLEMS.

69. Proposed by WILLIAM SYMMONDS, M. A., Professor of Mathematics and Astronomy in Pacific College, Santa Rosa, California ; P. O., Sebastopol, California.

To divide a square card into right-lined sections in a manner, that a rectangle of a given breadth can be formed from the sections; likewise, form a square from a rectangular card.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas.

(1). Let $ABCD$ be the square. Produce DA to H making AH equal the given width of the rectangle, join HB, and draw KO perpendicular to HB at its mid-point, then O is the center of the circle through HB. Produce AD to meet circle at G; AG is the length of the required rectangle. Take $AE=AH$ and complete the rectangle $AEFG$.



Now the right triangle AHB=right triangle BCN=right triangle MFG.

$\therefore$ $CN=AE$ and $DN=BE$;

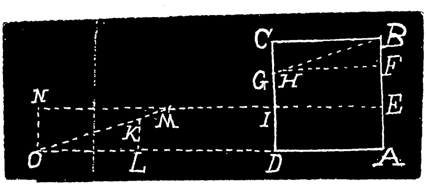
$\therefore$ $\triangle BEM=\triangle DNG$.

$\therefore$ $ABCD=ADNME+BCN+BEM$ $=ADNME+MFG+NDG=AEFG$.

(2). Let $AEFG$ be the given rectangle. Produce GA to H making $AH=AE$. Upon HG describe the semi-circle. Then AB is a side of the required square. Complete the square $ABCD$ and draw BG. The rest of the proof is the same as above.

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

(1). Let $ABCD$ represent the square card. From A lay off on AB the the width of the rectangle successively as many times as possible, as AE, EF.



Then from the opposite corner C, lay off *one width only* of the rectangle, as CG. Now cut through on line GB. Then cut FH and EI parallel to DA.